Period 4, November 22, 2024

$3^x = y$ Same

$\log_3 y = x$

$3^x = y$

Definition of the Logarithmic Function

For $x > 0$ and $b > 0, b \neq 1,$

base = t $\quad y = \log_b x$ is equivalent to $b^y = x$.

The function $f(x) = \log_b x$ is the **logarithmic function with base *b*.**

Location of Base and Exponent in Exponential and Logarithmic Forms

Exponent / Exponent

Logarithmic Form: $y = \log_b x$ $\Leftrightarrow$ Exponential Form: $b^y = x$

Base / Base

CHECK POINT 1 Write each equation in its equivalent exponential form:

a. $3 = \log_7 x$ **b.** $2 = \log_b 25$ **c.** $\log_4 26 = y.$

$7^3 = x$ $\quad b^2 = 25$ $\quad 4^y = 26$

$\log 7 = x \Rightarrow 10^x = 7$

no base, Then your base is 10

Write each equation in its equivalent logarithmic form:

a. $12^2 = x$ **b.** $b^3 = 8$ **c.** $e^y = 9.$

$\log_{12} x = 2$ $\quad \log_b 8 = 3$ $\quad \log_e 9 = y$ $\quad \log_e a = \ln a$

Evaluate:

a. $\log_2 16 = a$ **b.** $\log_7 \frac{1}{49} = b$ **c.** $\log_{25} 5 = c$ **d.** $\log_2 \sqrt[5]{2}. = d$

$2^a = 16$

$2^a = 2^4$

$a = 4$

$7^b = \frac{1}{49}$

$7^b = \frac{1}{7^2}$

$7^b = 7^{-2}$

$b = -2$

$25^c = 5 = \sqrt{25}$

$\sqrt{25} = 5$

$25^c = 25^{\frac{1}{2}}$

$\frac{1}{2} = c$

$2^d = \sqrt[5]{2} = 2^{\frac{1}{5}}$

$d = \frac{1}{5}$

Basic Logarithmic Properties Involving One

1. $\log_b b = 1$ because 1 is the exponent to which b must be raised to obtain b. $(b^1 = b)$
2. $\log_b 1 = 0$ because 0 is the exponent to which b must be raised to obtain 1. $(b^0 = 1)$

$\text{Log}_b b = c \Rightarrow b^c = b \Rightarrow b^c = b^1 \Rightarrow c = 1$

$\text{Log}_b 1 = c \Rightarrow b^c = 1 \Rightarrow b^c = b^0 \Rightarrow c = 0$

Evaluate:

a. $\log_7 7 = a$

$7^a = 7$

$a = 1$

b. $\log_5 1. = d$

$5^d = 1$

$5^d = 5^0$

$d = 0$

Inverse Properties of Logarithms

For $b > 0$ and $b \neq 1$,

$\log_b b^x = x$ The logarithm with base b of b raised to a power equals that power.

$b^{\log_b x} = x.$ b raised to the logarithm with base b of a number equals that number.

$\text{Log}_6 9 = b \Rightarrow 6^b = 9$

Evaluate:

a. $\log_4 4^5 = a = 5$

$4^a = 4^5$

$a = 5$

b. $6^{\log_6 9} = b$

$6^b = 9$

EXAMPLE 6 Graphs of Exponential and Logarithmic Functions

Graph $f(x) = 2^x$ and $g(x) = \log_2 x$ in the same rectangular coordinate system.

$y=2^x \Leftrightarrow \log_2 x = y \Rightarrow 2^y = x$

inverse

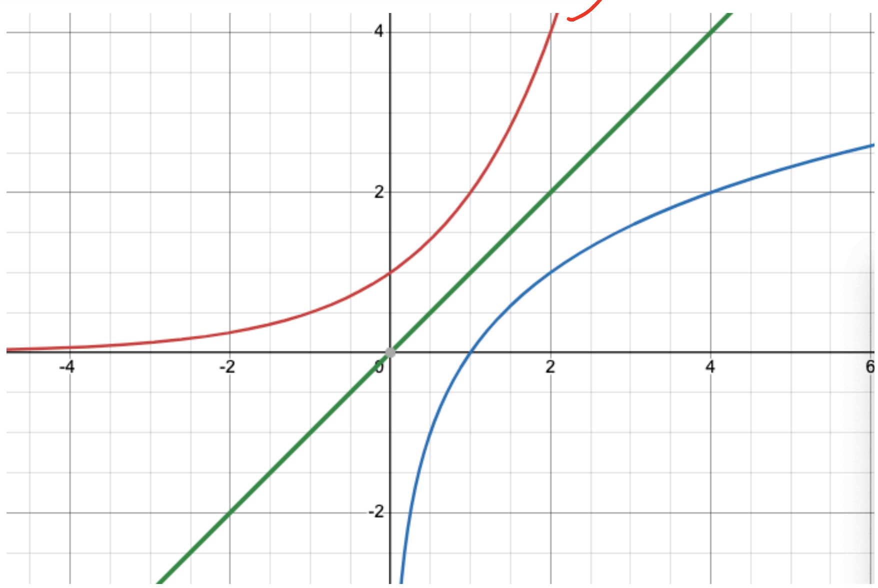
Characteristics of Logarithmic Functions of the Form $f(x) = \log_b x$

1. The domain of $f(x) = \log_b x$ consists of all positive real numbers: $(0, \infty)$. The range of $f(x) = \log_b x$ consists of all real numbers: $(-\infty, \infty)$.
2. The graphs of all logarithmic functions of the form $f(x) = \log_b x$ pass through the point $(1, 0)$ because $f(1) = \log_b 1 = 0$. The x-intercept is 1. There is no y-intercept.
3. If $b > 1$, $f(x) = \log_b x$ has a graph that goes up to the right and is an increasing function.
4. If $0 < b < 1$, $f(x) = \log_b x$ has a graph that goes down to the right and is a decreasing function.
5. The graph of $f(x) = \log_b x$ approaches, but does not touch, the y-axis. The y-axis, or $x = 0$, is a vertical asymptote.

$y = 2^x$ Domain $\mathbb{R}$ Range $y>0$

$x = 2^y$ Domain $x>0$ Range $\mathbb{R}$

$y = x$



Transformation	Equation	Description
Vertical translation	$g(x) = \log_b x + c$ $g(x) = \log_b x - c$	• Shifts the graph of $f(x) = \log_b x$ upward c units. • Shifts the graph of $f(x) = \log_b x$ downward c units.
Horizontal translation	$g(x) = \log_b(x + c)$ $g(x) = \log_b(x - c)$	• Shifts the graph of $f(x) = \log_b x$ to the left c units. Vertical asymptote: $x = -c$ • Shifts the graph of $f(x) = \log_b x$ to the right c units. Vertical asymptote: $x = c$
Reflection	$g(x) = -\log_b x$ $g(x) = \log_b(-x)$	• Reflects the graph of $f(x) = \log_b x$ about the x-axis. • Reflects the graph of $f(x) = \log_b x$ about the y-axis.
Vertical stretching or shrinking	$g(x) = c \log_b x$	• Vertically stretches the graph of $f(x) = \log_b x$ if $c > 1$. • Vertically shrinks the graph of $f(x) = \log_b x$ if $0 < c < 1$.
Horizontal stretching or shrinking	$g(x) = \log_b(cx)$	• Horizontally shrinks the graph of $f(x) = \log_b x$ if $c > 1$. • Horizontally stretches the graph of $f(x) = \log_b x$ if $0 < c < 1$.

EXAMPLE 8 **Modeling Height of Children**

The percentage of adult height attained by a boy who is x years old can be modeled by

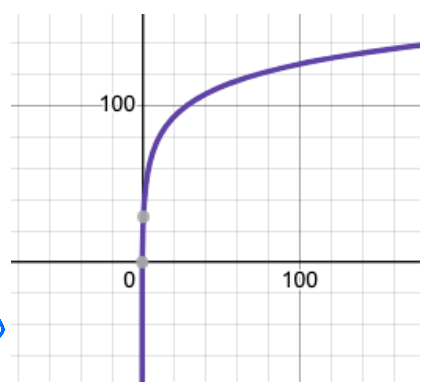
$$f(x) = 29 + 48.8 \log(x + 1),$$

where x represents the boy's age (from 5 to 15) and $f(x)$ represents the percentage of his adult height. Approximately what percentage of his adult height has a boy attained at age 8?

1

$$y = 29 + 48.8 \log(x + 1)$$

2



$F(8) = 29 + 48.8 \log(8+1)$

$F(8) = 29 + 48.8 \log 9 = 29 + 48.8(.95)$

$= 29 + 46.6$

$= 75.6\%$

Properties of Common Logarithms

General Properties	Common Logarithms
1. $\log_b 1 = 0$	**1.** $\log 1 = 0$
2. $\log_b b = 1$	**2.** $\log 10 = 1$
3. $\log_b b^x = x$	**3.** $\log 10^x = x$
4. $b^{\log_b x} = x$	**4.** $10^{\log x} = x$

Inverse properties

$\log_b a^c = c \log_b a$

$\log_b a^c = y$

$a^c = b^y$

$\frac{\cancel{c} \log_b a}{\cancel{c}} = \frac{y}{c}$

$\log_b a = \frac{y}{c}$

$(b^{\frac{y}{c}})^c = (a)^c$

$b^{\frac{y}{\cancel{c}}\cancel{c}} = a^c$

$b^y = a^c$

EXAMPLE 9 **Earthquake Intensity**

The magnitude, R, on the Richter scale of an earthquake of intensity I is given by

$$R = \log \frac{I}{I_0},$$

where I_0 is the intensity of a barely felt zero-level earthquake. The earthquake that destroyed San Francisco in 1906 was $10^{8.3}$ times as intense as a zero-level earthquake. What was its magnitude on the Richter scale?

$R = \log \frac{(10^{8.3})(\cancel{I_0})}{\cancel{I_0}} = \log 10^{8.3} = 8.3$

$8.3 \log_{10} 10 = 8.3(1) = 8.3$

$\log_{10} 10^{8.3} = a \Rightarrow 10^a = 10^{8.3} \Rightarrow a = 8.3$

natural logarithmic function. $f(x) = \ln x$, read "el en of x."

$\log_b a + \log_b c = \log_b ac$

$\log_b a - \log_b c = \log_b \frac{a}{c}$

Properties of Natural Logarithms

General Properties	Natural Logarithms
1. $\log_b 1 = 0$	**1.** $\ln 1 = 0$
2. $\log_b b = 1$	**2.** $\ln e = 1$
3. $\log_b b^x = x$	**3.** $\ln e^x = x$
4. $b^{\log_b x} = x$	**4.** $e^{\ln x} = x$

Inverse properties

Solve the given polynomial equation. Use the Rational Zero Theorem and Descartes's Rule of Signs as an aid in obtaining the first root.

$6x^3 - 7x^2 - 6x - 1 = 0$

6
3 2
6 1

$1, -1, \frac{1}{6}, -\frac{1}{6}, +\frac{1}{3}, -\frac{1}{3}, +\frac{1}{2}, -\frac{1}{2}$

$F(1) = 6(1)^3 - 7(1)^2 - 6(1) - 1 = 6 - 7 - 6 - 1 \neq 0$

$F(-1) = 6(-1)^3 - 7(-1)^2 - 6(-1) - 1 = -6 - 7 + 6 - 1 \neq 0$

$F(\frac{1}{6}) = 6(\frac{1}{6})^3 - 7(\frac{1}{6})^2 - 6(\frac{1}{6}) - 1$

$\frac{1}{36} - \frac{7}{36} - 1 - 1 \neq 0$

$F(-\frac{1}{6}) \neq 0$

$F(\frac{1}{3}) = 6(\frac{1}{3})^3 - 7(\frac{1}{3})^2 - 6(\frac{1}{3}) - 1 = \frac{6}{27} - \frac{7}{9} - \frac{6}{3} - 1 = \frac{2}{9} - \frac{7}{9} - \frac{18}{9} - \frac{9}{9}$

$F(-\frac{1}{3}) = 6(-\frac{1}{3})^3 - 7(-\frac{1}{3})^2 - 6(-\frac{1}{3}) - 1 = -\frac{2}{9} - \frac{7}{9} + \frac{18}{9} - \frac{9}{9} = 0$

$$
\begin{array}{r|rrrr}
-\frac{1}{3} & 6 & -7 & -6 & -1 \\
 & & -2 & 3 & 1 \\
\hline
 & 6 & -9 & -3 & \boxed{0}
\end{array}
$$

$a=2$
$b=-3$
$c=-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\left(x+\tfrac{1}{3}\right)\left(6x^2-9x-3\right)$$

$$\left(x+\tfrac{1}{3}\right)\cdot(3)\left(2x^2-3x-1\right)$$